AGNAGN semi
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Section 5.7 - End of Section 5

Observation of Ionized Gas in a Nebula

By observing the ionized gas in a nebula:

- determine the number of ionizing photons emitted by the central star.
- determine a long baseline color index
- estimate the effective temperature of the star and reconstruct its spectral energy distribution (SED).

1. Assumption: The nebula is optically thick to the Lyman continuum

Condition: optically thick to the Lyman continuum -> all photons capable of ionizing hydrogen are absorbed within the nebula.

-> The number of ionizing photons emitted by the star = The number of hydrogen ionizations occurring in the nebula per unit time.

2. When the nebula is in equilibrium

Ionized hydrogen atoms will eventually recombine with free electrons.

-> The ionization rate = The recombination rate (per unit time).

This relationship can be expressed with the following equation:

$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(\mathrm{H}^0) = \int_0^{r_1} n_p n_e \alpha_B(\mathrm{H}^0, T)\, dV,$$

Lv : the luminosity of the star per unit frequency interval

Left-hand side: Total number of ionizing photons emitted by the star per unit time

Right-hand side: Total number of recombinations occurring within the nebula per unit time

α_B : Case B recombination coefficient

3. Relation to a Specific Emission Line (e.g., Hβ)

- intensity of a specific emission line from the nebula: directly related to the recombination processes within the nebula.
- luminosity in the Hβ line, L(Hβ), can be calculated as:

$$L(\mathrm{H}\beta) = \int_0^{r_1} 4\pi j_{\mathrm{H}\beta}\, dV$$
$$= h\nu_{\mathrm{H}\beta} \int_0^{r_1} n_p n_e \alpha_{\mathrm{H}\beta}^{eff}(\mathrm{H}^0, T)\, dV$$

Here, $\alpha_{H\beta}^{eff}$: effective recombination coefficient for the Hβ transition

4. Proportional Relation Between Ionizing Photons and Hβ Photons

By dividing the two expressions derived in sections 2 and 3, we obtain:

$$\frac{\dfrac{L(\mathrm{H}\beta)}{h\nu_{\mathrm{H}\beta}}}{\displaystyle\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu} = \frac{\displaystyle\int_0^{r_1} n_p n_e \alpha_{\mathrm{H}\beta}^{eff}(\mathrm{H}^0, T)\, dV}{\displaystyle\int_0^{r_1} n_p n_e \alpha_B(\mathrm{H}^0, T)\, dV} \approx \frac{\alpha_{\mathrm{H}\beta}^{eff}(\mathrm{H}^0, T)}{\alpha_B(\mathrm{H}^0, T)} \tag{5.34}$$

- number of ionizing photons emitted by the star can be inferred from the observed luminosity of the Hβ line.
- only weakly dependent on the nebular temperature, and largely **independent of the nebular density structure** or homogeneity.

5. Ionizing Photon Count vs. Stellar Optical Brightness → Estimating Color Index
comparing the number of ionizing photons (in the ultraviolet) with the stellar luminosity $L_{\nu f}$ at some optical frequency:

$$\frac{L_{\nu_f}}{\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu} = \frac{L_{\nu_f}}{\frac{L(\mathrm{H}\beta)}{h\nu_{\mathrm{H}\beta}}} \frac{\frac{L(\mathrm{H}\beta)}{h\nu_{\mathrm{H}\beta}}}{\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu} = h\nu_{\mathrm{H}\beta} \frac{\alpha_{\mathrm{H}\beta}^{eff}(\mathrm{H}^0, T)}{\alpha_B(\mathrm{H}^0, T)} \frac{\pi F_{\nu_f}}{\pi F_{\mathrm{H}\beta}} \tag{5.35}$$

- ratio acts like a **color index**
- the **effective temperature of the star** can be estimated
- reason: hotter stars emit more ultraviolet (ionizing) photons.

6. Practical Advantages
This method offers the following advantages:
• the measurement is **independent of distance**.
•If both measurements are made at the **same wavelength** -> the effects of **interstellar extinction** are canceled out.

5.10 Ionizing Radiation from Stars

From this part:
- how to observationally estimate the amount of ultraviolet (ionizing) radiation emitted by a star
- how to use this to infer the star's effective temperature (T★)
- focusing particularly on the classical Zanstra method.

1. Use of Broad-Band vs. Narrow-Band Filters
- measuring stellar brightness -> broad-band filters commonly used
  - as they cover wide wavelength ranges.
- if the star is embedded in a **bright and complex nebula**,
  - often advantageous to use **narrow-band** filters that avoid strong nebular emission lines
  - and instead sample relatively clean parts of the stellar continuum spectrum.
    - For example, filters placed between the strong Hα and [O III] emission lines.

2. Basic Concept of the Zanstra Method

Zanstra method: estimates the amount of stellar ionizing ultraviolet radiation
- The intensity of a nebular recombination line
- With the optical brightness of the central star

From this comparison, one can derive the **Zanstra temperature (T_*)** of the star.
-> Zanstra originally approximated the **star's spectrum** as a **Planck function $B_\nu(T_*)$**

3. Why is the Planck Function Inaccurate?

actual stellar radiation spectrum differs significantly from a simple blackbody (Planck) spectrum.

Why?

Because complex physical processes occur in the **stellar atmosphere**.

For example:

•Near the Lyman limit (912 Å) or the ionization edges of He^{+} and He^{2+}

-> there are noticeable discontinuities ("jumps") and abrupt changes in opacity.

-> to accurately determine the amount of ionizing radiation emitted by a star, it is necessary to use **model stellar atmosphere spectra**, rather than a blackbody approximation (e.g., spectra like those shown in Figure 5.16).

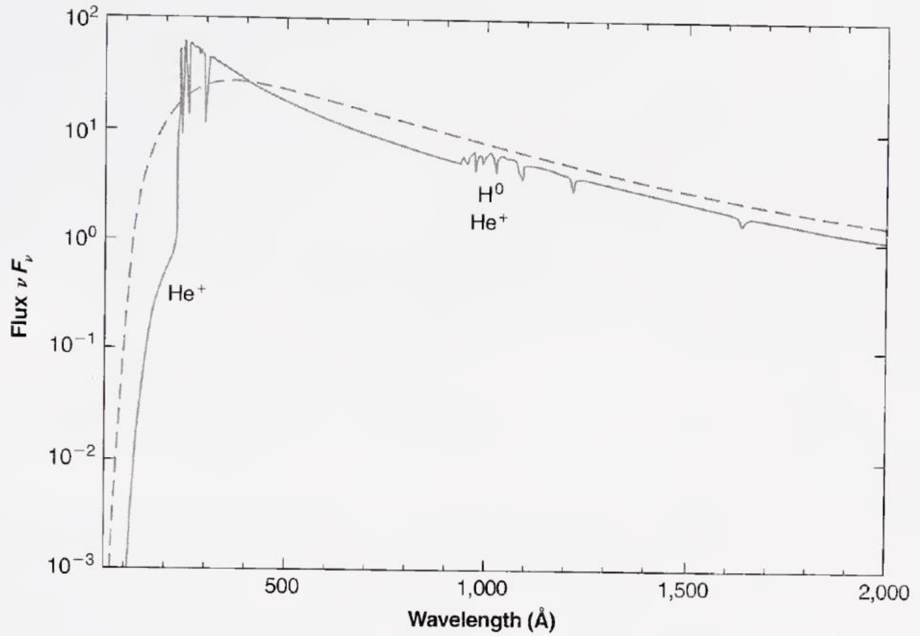

Figure 5.16

Calculated flux from a model planetary-nebula central star with $T_* = 100{,}000$ K, $\log g = 6$ (*solid line*), compared with blackbody flux for the same temperature (*dashed line*). The ionization edges of atomic hydrogen and the first ion of helium are marked.

4. He II 4686Å observation → Estimating UV from hotter stars

- To ionize He^{+}, the star must emit photons with hν > 54.4 eV
- Most O-type main sequence stars do not emit much energy at this level -> the He II 4686Å line is weak or absent.
- for central stars of planetary nebulae (very hot stars):
  - line clearly appears, and from it,
  - one can determine the number of He^{+}-ionizing photons the star emits.

$$\frac{L_{\nu_f}}{\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu} = \frac{\frac{L_{\nu_f}}{L(\mathrm{H}\beta)}}{\frac{h\nu_{\mathrm{H}\beta}}{\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu}} \frac{L(\mathrm{H}\beta)}{h\nu_{\mathrm{H}\beta}} = h\nu_{\mathrm{H}\beta} \frac{\alpha^{eff}_{\mathrm{H}\beta}(\mathrm{H}^0, T)}{\alpha_B(\mathrm{H}^0, T)} \frac{\pi F_{\nu_f}}{\pi F_{\mathrm{H}\beta}} \quad (5.35)$$

5. Two versions of Zanstra temperature: H I vs He II

T_{HI}^* : obtained through H I (e.g., Hβ)

T_{HeII}^* : obtained through He II 4686Å

→ It would be ideal if these two temperatures matched, but they often **do not**.

(Example: In NGC 7662, 70,000 K from H I vs. 113,000 K from He II — a large difference)

6. Cause of this temperature discrepancy: optical thickness of the nebula

- above calculations **assume:** nebula is **optically thick**.
- in reality, the nebula may be **density-bounded**.
  - not all ionizing photons are absorbed — some may escape.
- Number of ionizing photons emitted by the star ≠ Number of recombinations in the nebula
  → If estimate the number of ionizing photons only from Hβ emission -> underestimate it.

7. In such cases, the equation must be modified

When only a portion of the ionizing photons emitted by the star are absorbed by the nebula, Equation (5.35) must be modified as follows:

$$\frac{L_{\nu_f}}{\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu} = \eta_{\mathrm{H}} h\nu_{\mathrm{H}\beta} \frac{\alpha^{eff}_{\mathrm{H}\beta}(\mathrm{H}^0, T)\pi F_{\nu_f}}{\alpha_B(\mathrm{H}^0, T)\pi F_{\mathrm{H}\beta}}$$

η_H : the fraction of the H-ionizing photons that are absorbed in the nebula

8. Could He II also be density-bounded?

- Theoretically, it's possible,
- but in most planetary nebulae, this does not apply.
    - **Reason:** The presence of He I emission lines in the nebula implies that an outer He^+ zone exists.
    - This means the nebula is **optically thick** to He^+ ionizing radiation → $\boldsymbol{\eta_{He+} = 1}$

5.10 Ionizing Radiation from Stars

From this:
- a method for more accurately estimating the ultraviolet radiation and temperature of a star
- particularly the central star of a planetary nebula.
- **Goal**: To determine the star's effective temperature using the number of ultraviolet (ionizing) photons it emits

① What can be learned from He I emission lines:
→ The number of photons capable of ionizing neutral helium (He^0)
intensity of lines: to estimate the number of high-energy photons capable of ionizing He^0.
Precondition:
The nebula must be optically thick to He^0-ionizing radiation—
How to check if this condition is met:
- The He^+ region is smaller than the H^+ region, or
- The observed He^+/H^+ ratio is very small (e.g., < 0.1)

→ number of He^0-ionizing photons emitted by the star is equal to the number of He I photons produced by recombination.

② Comparison in actual observations (Zanstra temperature comparison)
Zanstra temperature is compared using two different methods:
One based on the **H I recombination line** (e.g., Hβ)
The other based on the **He II recombination line** (4686 Å)
→ In general, the **He II Zanstra temperature** is higher than the **H I Zanstra temperature**.

Reason:
The nebula is not completely opaque (optically thick) to hydrogen-ionizing radiation.
→ Some ionizing photons for hydrogen escape the nebula
→ using only Hβ underestimates the actual number of ionizing photons
→ He II-based temperature is more accurate

5.11 Abundances of the Elements in Nebulae

Main topic of this chapter:
- how to measure the elemental abundances in nebulae (such as H II regions and planetary nebulae)
- challenges and limitations of these measurements, and even recent debates on the topic.

1. How to determine elemental abundances in nebulae

Measure the relative intensity of emission lines

Nebulae are optically thin -> abundances can be estimated directly. -> No need to deal with complex concepts like the curve of growth
- In the optical range: H, He, N, O, Ne (but not carbon)
- In the UV/IR range: C, O^4, O^5, S, Ar, and others are also accessible

2. Types of emission lines and their temperature dependence

Recombination lines (e.g., Hβ, He I λ5876):
- Not sensitive to temperature → More stable and reliable
- However, very weak for heavy elements → Difficult to observe

Collisionally excited lines (CELs) (e.g., [O III] λ5007, [N II] λ6583):
- Highly temperature-dependent → Accurate knowledge of temperature is essential
- Many are bright and strong -> commonly used in abundance studies

3. Temperature Issue: Even within the same nebula, temperature is not uniform
Observed temperature differences:
Forbidden line ratios → Estimate higher temperatures
Recombination lines, free-free continuum, Balmer jump → Estimate lower temperatures

Why does this happen?
Forbidden lines: As temperature ↑ → line intensity ↑
Recombination lines: As temperature ↑ → line intensity ↓
➜ Therefore, the average temperature inferred depends on the measurement method
➜ Consequently, the derived abundances also vary

4. Methods that account for temperature distribution:
Simple model: Assumes uniform temperature and density
→ Fast, but may deviate from reality
Improved model: Takes temperature fluctuations into account

a range of temperatures exists -> introduce temperature variance t^2

By using two types of emission line ratios, both the mean temperature (T_0) and the variance (t^2) can be estimated together

5. Example of He/H Ratio Measurement (Orion Nebula)

•He I lines are detected, but He II lines are not → He^{+} is observed, but He^{0} is not

•If neutral helium (He^{0}) exists, the derived He^{+} abundance will be underestimated → correction is needed

•Use the intensity of S^{+} ions to estimate the amount of He^{0} (since both have similar ionization energies)

6. Measuring the Abundances of Metal Elements: Table 5.3

Comparison of the Sun, Orion Nebula (NGC 1976), and average planetary nebulae

Examples:

•O/H: Sun ~7.4×10^{-4}, H II region ~4×10^{-4}

•N/H: Sun ~9×10^{-5}, Planetary nebula ~2×10^{-4} (affected by nuclear processing in stars)

7. Current Issues and Debates

Abundances derived from forbidden lines and recombination lines differ!

•In some nebulae, the difference can be up to a factor of 10

•Small differences (50–100%) can be explained by temperature fluctuations (t^2)

•larger discrepancies remain unexplained → suggests the presence of unknown physical processes

Table 5.3

Abundances of the elements

N	Atom	Sun	H II Region	Planetary
1	H	1	1	1
2	He	0.1	0.095	0.10
6	C	3.5×10^{-4}	3×10^{-4}	8×10^{-4}
7	N	9.3×10^{-5}	7×10^{-5}	2×10^{-4}
8	O	7.4×10^{-4}	4×10^{-4}	4×10^{-4}
10	Ne	1.2×10^{-4}	6×10^{-5}	1×10^{-4}
11	Na	2.1×10^{-6}	3×10^{-7}	2×10^{-6}
12	Mg	3.8×10^{-5}	3×10^{-6}	2×10^{-6}
13	Al	2.9×10^{-6}	2×10^{-7}	3×10^{-7}
14	Si	3.6×10^{-5}	4×10^{-6}	1×10^{-5}
16	S	1.6×10^{-5}	1×10^{-5}	1×10^{-5}
17	Cl	1.9×10^{-7}	1×10^{-7}	2×10^{-7}
18	Ar	4.0×10^{-6}	3×10^{-6}	3×10^{-6}
19	K	1.3×10^{-7}	1×10^{-8}	1×10^{-7}
20	Ca	2.3×10^{-6}	2×10^{-8}	1×10^{-8}
26	Fe	3.2×10^{-5}	3×10^{-6}	5×10^{-7}

8. Results may vary depending on the measurement method

Method	Advantage	Disadvantage
Recombination lines	Less sensitive to temperature, relatively accurate	Weak lines, difficult to observe
Forbidden lines (CELs)	Strong lines, easy to measure	Sensitive to temperature, requires correction
Radio recombination lines	Can be used at large distances, less affected by extinction	Difficult to correct for neutral helium (He^0)

5.12 Calculations of the Structure of Model Nebulae

Key Point:
This section deals with **theoretical methods** for calculating the **structure** and **physical properties of nebulae**, especially H II regions and planetary nebulae.

Overall Goal: Build a Nebula Model
Why do we need a model?
- observe through a telescope is only the light (spectrum) emitted by a nebula.
- analyzing this light: can determine the temperature, density, ionization state, and even the properties of the central star within the nebula.
- The way we achieve this is through modeling — that is, mathematically simulating a real nebula.

Step 1: Set Assumptions
To construct a model, we first assume several physical conditions.
For the central star:
- Temperature (e.g., 100,000 K)
- Luminosity (e.g., 100 $L_\odot$)
- Spectral shape (e.g., blackbody or realistic stellar atmosphere model)

For the nebula:
- Density distribution (e.g., uniform or decreasing from the center)
- Elemental composition (e.g., H, He, O, N, etc.)
- Size and structure (often assumed to be spherical)

Step 2: Calculating the Nebular Structure with Equations
At this stage, physical equations come into play. These are the key formulas used to mathematically describe the **interior** of a nebula.

(1) Radiative Transfer Equation
calculates how much starlight is **absorbed** and how much is **re-emitted** as it passes through the nebula.

$$\frac{dI_\nu}{ds} = -\frac{d\tau_\nu}{ds} I_\nu + j_\nu = -\alpha_\nu I_\nu + j_\nu$$

I_ν: Intensity of the radiation
α_ν: Absorption coefficient (opacity)
j_ν: mission coefficient (emissivity)

(2) Ionization Equilibrium Equation
calculates the balance between **ionization** and **recombination** processes.

$$n(X^{+i}) \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(X^{+i}) d\nu = n(X^{+i}) n_e \alpha_G(X^{+i+1}, T)$$

Left side: Rate of ionization
Right side: Rate of recombination

3) Energy Balance Equation
calculates the condition where heating equals cooling:

Heating=Recombination cooling + Free-free cooling + Line cooling

Main cooling mechanism:
Collisionally excited lines — heavy elements like O, N, and S are primarily responsible for cooling.

Step 3: Perform Numerical Calculations
Using the equations above, a computer calculates the nebular properties **spatially**, yielding:
- Temperature at each position: T(r_1), T(r_2), T(r_3)
- Ionization structure (e.g., H^+, He^+, O^{2+}, etc.)
- Emission spectrum at each wavelength

Two calculation approaches:
1.On-the-spot approximation
  1. Assumes that ionizing photons from recombination are immediately absorbed nearby
  2. Fast and simple to compute

2.Full radiation field
  1. Considers all photon travel paths and interactions
  2. More accurate but computationally complex → requires iterations

5.12 Calculations of the Structure of Model Nebulae

Step 4: Compare with Observations → Refine the Model

results are compared with actual observations from telescopes:

- Spectral line intensities
- Ion distributions
- Temperature structure

If they don't match?

- Adjust the temperature of the central star
- Modify the density distribution
- Revise the elemental composition (e.g., metallicity)

→ The model is iteratively improved through repeated comparisons.