Logarithmic Rotation Curve and Multiple Bulges around the Galactic Center

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(Received ; accepted )

Abstract

We analyze a logarithmic rotation curve of the Galactic Center obtained from longitute-velocity diagrams in molecular lines. We show that the dynamical mass structure in the galactic bulge is deconvolved into multiple components: the inner core bulge and the main bulge. The entire Galactic mass structure is composed of the central black hole, inner (core) bulge, main bulge, exponential disk, and the dark halo. Both the inner and main bulges are represented by exponential spheroids, each of which has exponential density profile. The de Vaucouleurs law was found to fail in representing the dynamical mass distribution in the galactic bulge.

Key words: galaxies: Galactic Center — galaxies: mass — galaxies: the Galaxy — galaxies: rotation curve — galaxies: bulge

1. Introduction

The distribution of mass density around the Galactic Center between the central black hole and the disk is one of the last-remained problems in the rotation curve study of the Galaxy (Sofue 2012). We recently obtained an almost completely sampled rotation curve of the entire Galaxy, in which the circular velocities from the central black hole to the outer dark halo are plotted without a gap of data (Sofue 2013: Paper I). The rotation curve was drawn in logarithmic scaling in order to represent the innermost region at high accuracy, which we call the logarithmic rotation curve (LogRC).

The mass distribution in the Galactic Center within $\sim 10$ pc has been extensively studied in relation to the central black hole by analyzing stellar kinematics (Genzel et al. 1994, 2010; Ghez et al. 2005, 2008; Gillessen et al. 2009). On the other hand, the mass distribution from the outer bulge to dark halo has been analyzed in detail using the rotation curve (Sofue and Rubin 2001; Sofue et al. 2009; Sofue 2013).

The present paper aims at determining the mass structure inside the bulge within several hundred parsecs based on rotation curve method. We derive the fundamental mass structure by deconvolving the rotation curve into several components. We show that the classical de Vaucouleurs law may not be a good approximation to the density distribution in the bulge. We propose a new model, which requires multiple components within the galactic bulge.

The galactocentric distance and circular velocity of the Sun are taken to be $(R_0, V_0)=(8.0$ kpc, $200$ km s$^{-1}$). We also examine a case for $(R_0, V_0)=(8.0$ kpc, $238$ km s$^{-1}$) as obtained by recent VLBI measurements (Honma et al. 2012).

2. Rotation Curve around the Galactic Center

2.1. Observations

Figure 1 shows an observed logarithmic rotation curve (LogRC) of the entire Galaxy obtained by Sofue (2013, Paper I). The central part within a few hundred pc has been obtained by analyzing CO line longitude-velocity diagrams of the Galactic Center. The outer rotation curve at $r > 100$ pc is a result of compilation of circular velocities from the literature, which we called the grand rotation curve (Sofue et al. 2009; Sofue 2012). This grand rotation curve covers the entire Galaxy from the central black hole to the dark halo without a gap of data.

The rotation velocity inside the galactic bulge is presented in more precise way by LogRC than those in the current studies. Particularly, the rotation velocities between the black hole and inner disk has been resolved into some distinct features.

The main bulge component has a velocity peak at $r \approx 400$pc with $V=250$ km s$^{-1}$. It declines toward the center steeply, followed by a plateau-like hump at $r \sim 30 - 3$ pc. The plateau-like hump is then merged by the Keplerian rotation curve corresponding to the central black hole at $r \sim 2$ pc. Here we use a black hole mass of $M_{BH}=4 \times 10^6 M_\odot$, taking the mean of the recent values converted to the case for $R_0=8.0$ kpc, i.e., $2.6 - 4.4 \times 10^6 M_\odot$ (Genzel et al. 2000, 2010), $4.1 - 4.3 \times 10^6 M_\odot$ (Ghez et al. 2005), and $3.95 \times 10^6 M_\odot$ (Gillessen et al. 2009).

2.2. Broad velocity maximum by de Vaucouleurs law

We first compare the observations with the best-fit de Vaucouleurs law for the galactic bulge as obtained in our previous works (Sofue et al. 2009), which is shown by the dashed lines in figure 2. It is obvious that the model fit is not sufficient in the inner several hundred pc.
Fig. 1. The most completely sampled logarithmic rotation curve of the entire Galaxy (Sofue 2012).

Fig. 2. Top: Logarithmic rotation curve (LogRC) of the Galaxy compared with model curves. Solid lines represent the best-fit curve with two exponential-spherical bulges, exponential flat disk, and NFW dark halo. The classical de Vaucouleurs bulge is shown by dashed line, which is significantly displaced from the observation. Open circles are a new rotation curve adopting the recently determined circular velocity of the Sun, \( V_0 = 238 \) km s\(^{-1}\) (Honma et al. 2012). Bottom: Same, but the velocity scale is linear.
It is valuable to revisit de Vaucouleurs rotation curve, which is represented by \( \Sigma \propto e^{-(r/a)^{1/4}} \) with \( \Sigma \) and \( a \) being the surface mass density and scale radius. By definition scale radius \( a \) used here is equal to \( R_h/3460 \), where \( R_h \) is the half-surface mass radius used in usual de Vaucouleurs law expression (e.g. Sofue et al. 2009).

Since \( \Sigma \) is nearly constant at \( r \ll a \), the volume density varies as \( \Sigma \propto 1/r \) and the enclosed mass \( \propto r^2 \). This leads to circular velocity \( V = \sqrt{GM/r} \propto r^{1/2} \) near the center. Thus, the rotation velocity rises very steeply with infinite gradient at the center. It should be compared with the mildly rising velocity as \( V \propto r \) at the center in the other models.

At large \( r > a \), the de Vaucouleurs law has slower density decrease due to the weaker dependence on \( r \) \( (r^{1/4} \) effect) than the other models. This leads to more gentle decrease after the maximum. Figure 3 shows normalized behaviors of rotation velocity for de Vaucouleurs and other models. As the result of steeper rise near the center and slower decrease at large radii, the de Vaucouleurs rotation curve shows a much broader maximum in logarithmic plot compared to the other models.

We here define the half-maximum logarithmic velocity width by \( \Delta_{\log} = \log r_2 - \log r_1 \), where \( r_2 \) and \( r_1 \) \( (r_2 > r_1) \) are the radii at which the rotational velocity becomes equal to a half of the maximum velocity. From the calculated curves in figure 3, we obtain \( \Delta_{\log} = 3.0 \) for de Vaucouleurs, while \( \Delta_{\log} = 1.5 \) for the other models. Thus the de Vaucouleurs’s logarithmic curve width is twice the others, and the curve’s shape is much milder. Note that the logarithmic curve shape keeps the similarity against changed parameters such as the mass and scale radius.

In figure 2 (top) we show LogRC calculated for the de Vaucouleurs model by dashed lines and compare with the observations. It is obvious that the de Vaucouleurs law cannot reproduce the observations inside \( \sim 200 \) pc. Note that the shape of the curve is scaling free in the logarithmic plot. The de Vaucouleurs curve can be shifted in both directions by changing the total mass and scale radius, but the shape is kept same.

2.3. Exponential spheroid model

Since the de Vaucouleurs law was found to fail to fit the observed LogRC, we now try to represent the inner rotation curve inside \( \sim 100 \) pc by different models. We propose a new functional form for the central spheroidal component, which we call the exponential sphere model. In this model, the volume mass density \( \rho \) is represented by an exponential function of radius \( r \) with a scale radius \( a \) as

\[ \rho(r) = \rho_c e^{-r/a}. \]  

(1)

The mass involved within radius \( r \) is given by

\[ M(R) = M_0 F(x), \]

(2)

where \( x = r/a \) and

\[ F(x) = 1 - e^{-x}(1 + x + x^2/2). \]

(3)

The total mass is given by

\[ M_0 = \int_0^\infty 4\pi r^2 \rho dr = 8\pi a^3 \rho_c. \]

(4)

The circular rotation velocity is then calculated by

\[ V(r) = \sqrt{GM/r} = \sqrt{GM_0 a F \left( \frac{r}{a} \right)} \]

(5)

where \( G \) is the gravitational constant.

This model is simpler than the current bulge models such as the de Vaucouleurs profiles. Since the density decreases faster, the rotational velocity has narrower peak near the characteristic radius in logarithmic plot as shown in figure 3. The exponential-sphere model is nearly identical to that for the Plummer’s law, and the rotation curves have almost identical profiles. Hence, the results in the present paper may not be much changed, even if we adopt the Plummer potential.

3. Model Fitting

3.1. Deconvolution of rotation curve

In order to fit the observed rotation curve by models, we assume the following components:

- The central black hole with mass \( M_{BH} = 4 \times 10^6 M_\odot \).
An innermost spheroidal component with the exponential-sphere density profile, or a central massive core.

• A spheroidal bulge with the exponential-sphere density profile.

• An exponential flat disk.

• A dark halo with NFW profile.

The approximate parameters for the disk and dark halo are adopted from the current study such as by Sofue (2012), and were adjusted here in order to better fit the data. The inner two spheroidal components were fitted to the data by trial and error by changing the parameter values.

After a number of trials, we obtained the best-fit parameters as listed in table 1. Figure 2 shows the calculated rotation curve for these parameters. The result satisfactorily represents the entire rotation curve from the central black hole to the outer dark halo.

We find that the fitting is fairly good in the Galactic Center, and the inner two peaks of rotation curve at \( r \sim 0.01 \text{ kpc} \) and \( \sim 0.5 \text{ kpc} \) are well reproduced by the two exponential spheroids. The figure also demonstrates that the present model is better than the de Vaucouleurs model shown by the dashed line. Figure 4 shows the usual presentation of the rotation curve up to 15 kpc in linear scales. The bottom panel enlarges the central several hundred pc region.

Table 1 lists the fitted parameters for individual components. The disk and halo parameters are about the same as those determined in our earlier paper (Sofue 2012). The classical bulge is composed of two superposed components. The inner bulge, or a massive core, has a mass of \( 5 \times 10^7 M_\odot \), scale radius of 3.8 pc, and the central density of \( 4 \times 10^4 M_\odot \text{pc}^{-3} \). The main bulge has a mass \( 10^{10} M_\odot \), scale radius 120 pc, and central density \( 200 M_\odot \text{pc}^{-3} \). The central volume densities are consistent with the surface mass density (SMD) of the order of \( \sim 10^5 M_\odot \text{pc}^{-2} \) at \( r \sim 3 \sim 10 \text{ pc} \) directly calculated from the rotation curve (Paper I).

### 3.2. Volume density

Figure 5 shows the resulting volume density profiles in the entire Galaxy for the total and individual mass components as functions of radius in logarithmic presentation. The bottom panel shows the same but for the Galactic Center in semi-logarithmic scaling.

The calculated dark matter distribution shows a steep cusp near the nucleus because of the \( 1/x \) factor in the NFW profile. However, since the functional form was derived from numerical simulations with much broader resolution (Navarro et al. 1996), the exact behavior in the immediate vicinity of the nucleus may not be taken so seriously, but it may include significant uncertainties.

### 3.3. Surface mass density

Using the best-fit model rotation curve, we calculated the surface-mass density (SMD) as a function of radius. Figure 6 shows the calculated results, both for the spherical and flat-disk assumptions by applying the method developed by Takamiya and Sofue (2000). In the figure, we also show the SMD distributions directly calculated using the observed rotation curve. The observed SMD is thus reproduced by the present model within errors of a factor of \( \sim 1.5 \) throughout the Galaxy. Here, the errors were estimated by eyes from the plots in the figure.

### 4. Discussion

We have fitted the most completely sampled LogRC of the Galaxy from the nucleus to the dark halo by a model assuming five mass components: the central black hole, inner spheroidal bulge (core) with exponential density profile, main spheroidal bulge with exponential profile, exponential flat disk, and dark halo with NFW density profile.

#### 4.1. Main bulge of exponential spheroid: Failure in de Vaucouleurs law

The classical de Vaucouleurs law was found to fail to fit the observations (figure 2). This fact was recognized for the first time by using the logarithmic rotation curve. As argued in section 2, the de Vaucouleurs profile for the sur-
Table 1. Parameters for exponential-sphere model of the bulge.

<table>
<thead>
<tr>
<th>Mass component</th>
<th>Total mass ($M_\odot$)</th>
<th>Scale radius (kpc)</th>
<th>Center density ($M_\odot$ pc$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black hole</td>
<td>4E6</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Inner bulge (core)</td>
<td>5.0E7</td>
<td>0.0038</td>
<td>3.6E4</td>
</tr>
<tr>
<td>Main Bulge</td>
<td>8.4E9</td>
<td>0.12</td>
<td>1.9E2</td>
</tr>
<tr>
<td>Disk</td>
<td>4.4E10</td>
<td>3.0</td>
<td>15</td>
</tr>
<tr>
<td>Dark halo (r ≤ h)</td>
<td>5E10</td>
<td>h = 12.0</td>
<td>$\rho_0 = 0.011$</td>
</tr>
</tbody>
</table>

face mass density requires a central cusp, yielding steeply rising circular velocity as $V \propto r^{1/2}$ at the center. Beyond the velocity maximum at $r > a$, it declines more slowly due to the extended outskirt. Thus, the logarithmic half-maximum velocity width is about twice that for the exponential spheroid or the Plummer law as shown in figure 3. This profile was found to be inappropriate to reproduce the observations as indicated in figure 2.

In order to reproduce the observations, we proposed a new bulge model, in which the volume density is represented by a simple exponential function as $\rho = \rho_0 e^{-r/a}$. The main bulge was found to be represented well by the exponential-spheroid model with mass $8 \times 10^9 M_\odot$ and scale radius 120 pc as shown in figures 2 and 4.

4.2. Inner bulge (core) — Dynamical link to the black hole

Inside the main bulge, significant excess of rotation velocity was observed over those due to the black hole and
main bulge (figures 1 and 2). This component was well explained by an additional inner spheroidal bulge of a mass of $\sim 5 \times 10^7 M_\odot$ with the same exponential density profile as the main bulge with scale radius 3.8 pc.

Considering the relatively large scatter and error of data at $r \sim 3-20$ pc, the density profile may not be strictly conclusive. However, the velocity excess should be taken as the evidence for existence of an additional mass component filling the space between the black hole and main bulge, which we called the inner bulge. As an alternative mass model to explain the plateau-like velocity excess, an isothermal sphere with flat rotation might be a candidate. However, it yields constant velocity from the center to halo, so that some artificial cut off of the sphere is required at some radius. Such a sphere with an artificial boundary may not be a good model for the Galaxy.

The Keplerian velocity by the central black hole of mass $4 \times 10^6 M_\odot$ declines to 100 km s$^{-1}$ at $r = 1.5$ pc, where the observed velocities are about the same. This implies that the mass of the inner bulge enclosed in this radius is negligible compared to the black hole mass. In fact, the present model indicates that the mass inside $r = 1$ pc is only $\sim 1.2 \times 10^5 M_\odot$, an order of magnitude smaller than the black hole mass.

The central $\sim 1$ pc region is, therefore, controlled by the strong gravity of the massive black hole. Stars there can no longer remain as a gravitationally bound system, but are orbiting around the black hole individually by Keplerian law. As an ensemble of stars orbiting the black hole may show velocity dispersion on the order of $v_\sigma(r) \sim 125(r/1\text{pc})^{-1/2}\text{km s}^{-1}$.

4.3. Enclosed mass – Comparison with results from stellar dynamics

The present model may be compared with the measurements of enclosed mass using stellar dynamics by Genzel et al. (1994), which have been obtained using several kinds of objects such as giant stars, He I stars, HI/CO gases, circumnuclear disk, and mini spirals (See the literature for details).

Figure 7 shows the enclosed mass as a function of radius calculated by the present model. In the figure we overlay the results by Genzel et al. (1994). Thereby, the data have been converted to the case of $R_0 = 8.0$ kpc from 8.5 kpc as adopted in their paper. Namely, the radius scale was multiplied by 8.0/8.5=0.94, and the mass scale was also multiplied by the same amount, because the mass is proportional to $\propto r^2$ with $r$ being the observed radial velocities.

The present model is in good agreement with the observed result by stellar dynamics. It should be stressed that the wavy variation of the model profile due to the multiple bulge structure is indeed observed by the stellar kinematics methods.

We comment on the general behavior of enclosed mass profiles against radius in the present model shown in figure 7. The enclosed mass for the black hole is trivially constant. The inner and main bulges have constant density near the center, which yields enclosed mass approximately proportional to $\propto r^3$ after volume integration. The disk model has constant surface density near the center, yielding enclosed mass $\propto r^2$ for surface integration. However, it would be much less because of the finite thickness for the real galactic disk. The NFW model shows a density cusp near the center as $\propto r^{-1}$, yielding enclosed mass proportional to $\propto r^2$. However, it should not be taken seriously because of the unknown accuracy of the model at these scales.

4.4. Effect of bars

As discussed in Paper I, we have abstracted circular velocities using observed longitude-velocity (LV) diagrams in the molecular lines. It was shown that the LV diagrams are influenced by some non-circular motions of the order of $\pm 20-30\%$ of the circular velocity. This may yield systematic errors of $\sim 40-60\%$ of mass estimation. Although the present result nicely represent the mass structure in the Galactic Center, the accuracy should be about $\pm 60\%$, or within a factor of $\sim 1.6$. However, this accuracy would be sufficient for examining the fundamental mass structure inside the bulge. In fact errors of factor $\sim 1.6$ appear relatively small in the logarithmic plots in figures 6 covering large dynamical ranges of order of six in both the SMD and radius axes.

One may wonder if the wavy profiles in the density and mass distributions in figures 5 and mass are due to velocity anomaly caused by such non-circular motions by a bar of a few pc scale. However, it is not clear whether a bar can be maintained in such a strong spheroidal gravity of the central massive black hole.

The agreement of the present analysis based on the cir-
cular rotation assumption with those from the spherical symmetric stellar dynamics, as shown in figure 7, may indicate that the bar’s effect will not be so significant in the central several tens of parsecs. Stellar bar dynamics and stability analysis in the close vicinity of the massive black hole, starting from a given mass structure such as that obtained here as the first approximation, would be an interesting subject for the future.

4.5 Correction for the solar velocity

Throughout the paper, we adopted the usual set of galactic parameters, \((R_0, \theta_0) = (8.0, 200)\) (kpc, km s\(^{-1}\)). In their recent trigonometric measurements of positions and velocities using VERA, Honma et al. (2012) obtained a faster circular velocity of the Sun of \(V_0 = 238\) km s\(^{-1}\).

If we adopt this new value, the general rotation velocities also increases by several to \(\sim 20\%\) in the outer disk. We have corrected the observed rotational velocities for the difference between the new and current circular velocities of the Sun, \(\Delta V = 238 - 200 = 38\) km s\(^{-1}\), using the following equation.

\[ V_c(r) = V(r) + \Delta V \left( \frac{r}{R_0} \right). \tag{6} \]

For globular clusters and satellite galaxies, the rotation velocities were obtained by multiplying \(\sqrt{2}\) to their radial velocities to yield expected Virial velocities. We have not applied the above correction to these cases, because their radial velocities are influenced only statistically by the change of solar velocity, and their mean values do not significantly change by different \(V_0\).

In figure 2, we plot the newly determined corrected rotation curve by open circles. The curve is not significantly changed in the central region as the above equation indicates. The outer most halo rotation curve using satellite galaxies also remains almost unchanged. A large difference is observed at \(r \sim 6\) to 20 kpc, where the rotation velocity is no longer flat. The rotation velocity increases up to \(\sim 20\) kpc, attaining a maximum at \(V \approx 270\) kpc. Beyond \(\sim 20\) kpc, the velocity decreases with radius at a slope roughly proportional to \(\propto r^{-1/2}\). This may imply that the dark halo is almost empty, and the NFW profile is no more good approximation.

Beyond \(r \sim 20\) kpc, the new rotation curve \(V_c\) declines more steeply than the curve calculated for the NFW profile. This suggests that the dark halo is more empty beyond \(\sim 20\) kpc than what obtained in the current data (Sofue 2012). It may indicate that the dark mass is more strongly concentrated within a radius \(\sim 30\) kpc. If this is the fact, the Galaxy is more isolated from the neighboring galaxies in so far as the dark matter distribution is concerned.

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